RADLEY COLLEGE
Entrance Scholarships

MATHEMATICS II

March 2009

Time allowed 1 hour

Show all working.

You may use a calculator

1. When you buy a new car, VAT is added at 15%. This means that a car with a pre VAT price of £10,000 actually costs £11,500.

(a) How much will you pay for a new car with a pre VAT price of £12,000?

A second new car costs £17,480 including VAT.

(b) How much would it cost pre VAT?

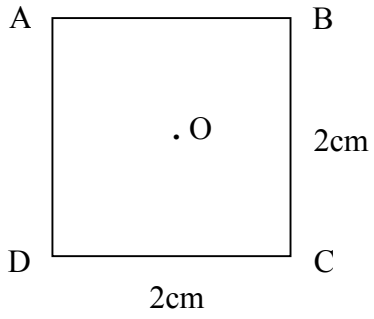
Last year when VAT was 17.5%, a third new car would have cost £21,620 including VAT.

(c) What does this new car cost this year, now that VAT is 15%?

(d) As a result of the change in VAT from 17.5% to 15% I save £540 on the cost of a fourth new car. How much do I pay for this car this year?

2. You will get extra marks for this question if you answer it without using a calculator.

(a)

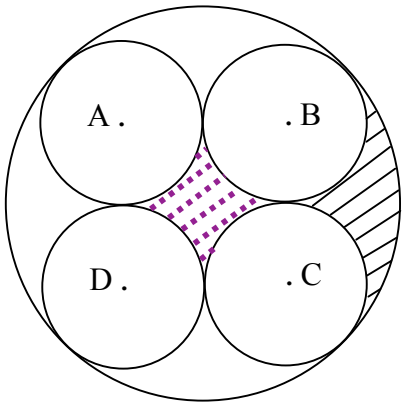


The diagram shows a square, ABCD, of centre O, and sides length 2cm.

(i) Show that AC= $\sqrt{8}$ cm

(ii) Show that AO= $\sqrt{2}$ cm

(b)



Four discs, each of radius 1cm, sit on a circular tray, as shown above.

(i) Calculate the radius of the tray

(ii) Show that the dotted shaded region in the middle of the tray has area $(4-\pi)cm^2$

(iii) Calculate the area of the striped shaded region which is to the right of discs B and C

3. Show that $x=5$, $y=-3$, is a solution for the pair of simultaneous equations

$$1.859x+2.108y=2.971$$
$$3.514x+4.596y=3.782$$

Hence, without doing any more working, write down the solution to each of the following pairs of simultaneous equations.

(a) $1.859x-2.108y=2.971$
$3.514x-4.596y=3.782$

(b) $2.108x+1.859y=2.971$
$4.596x+3.514y=3.782$

(c) $1859x+2108y=2971$
$3514x+4596y=3782$

(d) $1859x+2108y=2.971$
$3514x+4596y=3.782$

4. For any positive integer, n, we use the notation n! to denote the product of all of the positive integers between n and 1 inclusive. So, for example, $3! = 3\times 2\times 1 = 6$, and $5! = 5\times 4\times 3\times 2\times 1 = 120$

(a) Without using a calculator, and showing all of your working, find each of the following:

(i) $4!$

(ii) $1!$

(iii) $\dfrac{30!}{29!}$

(iv) $\dfrac{41!}{39!}$

(v) $\dfrac{101!}{98!}$

(b) Given $\dfrac{n!}{(n-2)!} = 12$

Deduce that $n^2 - n - 12 = 0$, and hence find the value of n.

5. (a) Calculate $\dfrac{1}{\frac{1}{2}-\frac{1}{3}}$

(b) Calculate $\dfrac{1}{\frac{1}{3}-\frac{1}{4}}$

(c) Calculate $\dfrac{1}{\frac{1}{4}-\frac{1}{5}}$

(d) Calculate $\dfrac{1}{\frac{1}{99}-\frac{1}{100}}$

(e) Write down a formula which summarises all of the above calculations

(f) Justify your answer.

6. (a) In a family of six children there are three sets of identical twins. They want to play a game that requires them to split into two teams, each team containing three children. They want to do this so that no team contains a pair of identical twins. How many ways are there of splitting the six children into teams?

(b) They now want to play another game that requires them to split into three teams, each team containing two children. Again they want to do this so that no team contains a pair of identical twins. How many different ways are there of splitting the six children this time?